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Selection rules for polymers and quasi-one-dimensional crystals: IV. Kronecker products for the line groups isogonal to D_{nh}

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Abstract. The reduction coefficients for the Kronecker products of irreducible representations are tabulated here for all the line groups isogonal to D_{nh} ($n = 1, 2, \ldots$) point groups. The derivation of the selection rules for the line groups, which describe spatial symmetries of polymers and quasi-one-dimensional solids, is thus completed. The results are interpreted as the conservation laws for certain quantum numbers (quasi-momentum, quasi-angular momentum, mirror-plane parities, etc).

1. Introduction

Conducting polymers and quasi-one-dimensional solids are undoubtedly one of the focal points of research in solid state physics today (Epstein and Conwell 1981, Miller 1982). Interest has thus arisen in studying their symmetry properties, and in particular in deriving the selection rules for various scattering processes in these systems. The spatial symmetries of polymers are described by the line groups; one therefore needs the reduction coefficients for the Kronecker products of the irreducible representations (reps) of the line groups. These coefficients have been tabulated in Damnjanović *et al* (1983, 1984) and Božović *et al* (1984) and in this paper, in which the last remaining family of line groups, those isogonal to D_{nh} ($n = 1, 2, \ldots$) point groups, is considered. Notice, however, that $D_{nh} = C_{nv} + \sigma_h C_{nv}$ and $C_{nv} = C_n + \sigma_v C_n$ (here C_n is the group generated by a rotation through $2\pi/n$ around the z axis, and σ_v and σ_h are the reflections in the xz and xy planes respectively) so that the line groups under study are two-step extensions. The number of reps to be considered is thus relatively large, and we have to use the short rep symbols and a number of other abbreviations; these are defined in § 2.

2. Abbreviations utilised throughout the paper

Expressions involving quasi-momenta (k, k') and quasi-angular momenta (m, m', j, j', h, q):

$$x = k - k', \qquad \text{where } k - k' \in (0, \pi)$$

$$y=\begin{cases} k+k', & \text{if } k+k'\in(0,\pi) \\ 2\pi-k-k', & \text{if } k+k'\in(\pi,2\pi) \end{cases}$$

$$z=\pi-k \qquad u=q-m$$

$$v=\begin{cases} m'-m, & \text{if } m-m'\in[\tfrac{1}{2}(3-n),-1] \\ m-m', & \text{if } m-m'\in[1,\tfrac{1}{2}(n-3)] \end{cases}$$

$$w=\begin{cases} m+m', & \text{if } m+m'\in[2,\tfrac{1}{2}(n-1)] \\ n-m-m', & \text{if } m+m'\in[\tfrac{1}{2}(n+1),n-1] \end{cases}$$

$$p=\begin{cases} m, & \text{if } m\in[1,h) \\ q-m, & \text{if } m\in(h,q) \end{cases}$$

$$p_1=\begin{cases} m+m', & \text{if } m+m'\in[1,h) \\ q-m-m', & \text{if } m+m'\in(h,q) \\ m-m'-n, & \text{if } m+m'\in(q,3h) \\ 3q-m-m', & \text{if } m+m'\in(3h,n) \end{cases}$$

$$p_2=\begin{cases} m-m', & \text{if } m-m'\in[1,h) \\ q-m+m', & \text{if } m-m'\in(h,q) \\ -m+m', & \text{if } m-m'\in(-h,1) \\ m-m'+q, & \text{if } m-m'\in(-q,h) \end{cases}$$

$$\beta=\begin{cases} q+j-m, & \text{if } j-m\in[2-q,-\tfrac{1}{2}(1+q)] \\ m-j, & \text{if } j-m\in[\tfrac{1}{2}(1-q),-1] \\ j-m, & \text{if } j-m\in[1,\tfrac{1}{2}(q-3)] \end{cases}$$

$$\gamma=\begin{cases} j+m, & \text{if } j+m\in[2,\tfrac{1}{2}(q-1)] \\ q-j-m, & \text{if } j+m\in[\tfrac{1}{2}(q+1),q-1] \\ j+m-2, & \text{if } j+m\in[q+1,\tfrac{3}{2}(q-1)] \end{cases}$$

$$\delta=\begin{cases} m-j, & \text{if } j-m\in[2-q,-1] \\ j-m, & \text{if } j-m\in[1,\tfrac{1}{2}(q-3)] \end{cases}$$

$$\varepsilon=\begin{cases} j+m, & \text{if } j+m\in[2,q-1] \\ 2q-j-m, & \text{if } j+m\in[q+1,\tfrac{3}{2}(q-1)] \end{cases}$$

$$\eta=\begin{cases} j-m+q, & \text{if } j-m\in[2-q,-1] \\ q+m-j, & \text{if } j-m\in[1,\tfrac{1}{2}(q-3)] \end{cases}$$

$$\zeta=\begin{cases} q-j-m, & \text{if } j+m\in[2,q-1] \\ j+m-q, & \text{if } j+m\in[q+1,\tfrac{3}{2}(q-1)] \end{cases}$$

$$\varkappa=q-j \qquad \mu=j+j' \qquad \varphi=q-j-j'$$

$$\lambda=\begin{cases} j'-j, & \text{if } j-j'\in[\tfrac{1}{2}(3-q),-1] \\ j-j', & \text{if } j-j'\in[1,\tfrac{1}{2}(q-3)] \end{cases}$$

$$\nu=\begin{cases} q+j-j', & \text{if } j-j'\in[\tfrac{1}{2}(3-q),-1] \\ q-j+j', & \text{if } j-j'\in[1,\tfrac{1}{2}(q-3)] \end{cases}$$

$$\rho = \begin{cases} h-m, & \text{if } m \in [1, h-1] \\ m-h, & \text{if } m \in [h+1, 2h+1] \end{cases}$$

$$\sigma = \begin{cases} h+m, & \text{if } m \in [1, h-1] \\ 3h-m, & \text{if } m \in [h+1, 2h-1] \end{cases}$$

$$\tau = h+j \qquad \psi = h-j.$$

Statements involving k, k', m, m' abbreviated by numbers or letters:

$(1) \leftrightarrow (m-m' \neq 0 \text{ and } m+m' \neq q)$ $(2) \leftrightarrow (m-m' = 0 \text{ and } m+m' \neq q)$
$(3) \leftrightarrow (m-m' \neq 0 \text{ and } m+m' = q)$ $(4) \leftrightarrow (m-m' = 0 \text{ and } m+m' = q)$.

Similarly,

$(a) \leftrightarrow (k-k' > 0 \text{ and } k+k' \neq \pi)$ $(b) \leftrightarrow (k-k' = 0 \text{ and } k+k' \neq \pi)$
$(c) \leftrightarrow (k-k' > 0 \text{ and } k+k' = \pi)$ $(d) \leftrightarrow (k-k' = 0 \text{ and } k+k' = \pi)$
$(e) \leftrightarrow (k-k' > 0 \text{ and } k+k' < \pi)$ $(f) \leftrightarrow (k-k' > 0 \text{ and } k+k' > \pi)$
$(g) \leftrightarrow (k-k' = 0 \text{ and } k+k' < \pi)$ $(h) \leftrightarrow (k-k' = 0 \text{ and } k+k' > \pi)$.

The conjunction of the statements (1) and (a) above is denoted by (a1), etc.

3. Results

The line groups isogonal to D_{nh} are $L(\overline{2n})2m$ ($n = 1, 3, \ldots$), Ln/mmm ($n = 2, 4, \ldots$), $L(\overline{2n})2c$ ($n = 1, 3, \ldots$), Ln/mcc ($n = 2, 4, \ldots$) and $L(2q)_q mcm$ ($n = 2q = 2, 4, \ldots$); the first two families contain symmorphic line groups and the latter families contain non-symmorphic ones. For each family we first give the character table to define the rep symbols and to specify the range of the labels (for more details cf Božović and Vujičić 1981), and then for every pair D, D' of reps we give the decompositon of the Kronecker product $D \times D'$. Since $D \times D' \sim D' \times D$, the tables are triangular. Each row is identified by a bold-faced rep symbol.

3.1. The symmorphic line groups $L(\overline{2n})2m$ ($n = 1, 3, \ldots$) and Ln/mmm ($n = 2, 4, \ldots$)

Table 1. The characters of the reps of the line groups $L(\overline{2n})2m$ ($n = 1, 3, \ldots$) and Ln/mmm ($n = 2, 4, \ldots$). Here $s = 0, 1, \ldots, n-1$; $t = 0, \pm 1, \ldots$; $\alpha = 2\pi/n$; the translation period is taken for the length unit and hence $k \in (0, \pi)$; m takes on all integral values from the interval $[1, \frac{1}{2}(n-1)]$; $q = n/2$. The four-dimensional reps appear only for $n \geq 3$. The short rep symbols are defined in the first column.

Rep	$(C_n^s \mid t)$	$(\sigma_v C_n^s \mid t)$	$(\sigma_h C_n^s \mid -t)$	$(\sigma_h \sigma_v C_n^s \mid -t)$
$(_0A_0\pm) = {}_0A_0^{\pm}$	1	1	± 1	± 1
$(_0B_0\pm) = {}_0B_0^{\pm}$	1	-1	± 1	∓ 1
$(_0Em\pm) = {}_0E_{m,-n}^{\pm}$	$2 \cos ms\alpha$	0	$\pm 2 \cos ms\alpha$	0
$(kEA_0) = {}_k^{-k}E_{A_0}$	$2 \cos kt$	$2 \cos kt$	0	0
$(kEB_0) = {}_k^{-k}E_{B_0}$	$2 \cos kt$	$-2 \cos kt$	0	0
$(kGm) = {}_k^{-k}G_{m,-m}$	$4 \cos kt \cos ms\alpha$	0	0	0
$(\pi A_0\pm) = {}_\pi A_0^{\pm}$	$(-1)^t$	$(-1)^t$	$\pm(-1)^t$	$\pm(-1)^t$
$(\pi B_0\pm) = {}_\pi B_0^{\pm}$	$(-1)^t$	$-(-1)^t$	$\pm(-1)^t$	$\mp(-1)^t$
$(\pi Em\pm) = {}_\pi E_{m,-m}^{\pm}$	$(-1)^t 2 \cos ms\alpha$	0	$\pm(-1)^t 2 \cos ms\alpha$	0

Table 1. (continued)

Rep	$(C_n^s \mid t)$	$(\sigma_v C_n^s \mid t)$	$(\sigma_h C_n^s \mid -t)$	$(\sigma_h \sigma_v C_n^s \mid -t)$
and only for $n = 2q = 2, 4, \ldots$				
$(0Aq\pm) = {}_0A_0^{\pm}$	$(-1)^s$	$(-1)^s$	$\pm(-1)^s$	$\pm(-1)^s$
$(0Bq\pm) = {}_0B_q^{\pm}$	$(-1)^s$	$-(-1)^s$	$\pm(-1)^s$	$\mp(-1)^s$
$(kEAq) = {}_k^{-k}E_{A_q}$	$(-1)^s 2\cos kt$	$(-1)^s 2\cos kt$	0	0
$(kEBq) = {}_k^{-k}E_{B_q}$	$(-1)^s 2\cos kt$	$-(-)^s 2\cos kt$	0	0
$(\pi Aq\pm) = {}_\pi A_q^{\pm}$	$(-1)^{s+t}$	$(-1)^{s+t}$	$\pm(-1)^{s+t}$	$\pm(-1)^{s+t}$
$(\pi Bq\pm) = {}_\pi B_q^{\pm}$	$(-1)^{s+t}$	$-(-1)^{s+t}$	$\pm(-1)^{s+t}$	$\mp(-1)^{s+t}$

Table 2. Decompositions of the Kronecker products of reps of $L(\overline{2n})2m$ (n odd) and Ln/mmm (n even). The rep symbols are defined in table 1.

(0A0+) × (0A0+) = (0A0+)
(0A0−) × (0A0±) = (0A0±)
(0B0+) × (0A0±) = (0B0±)   (0B0+) × (0B0+) = (0A0+)
(0B0−) × (0A0±) = (0B0∓)   (0B0−) × (0B0±) = (0A0∓)
(0Em+) × (0A0±) = (0Em+) × (0B0±) = (0Em+)
(0Em+) × (0Em′+) = (1) (0Ev+) + (0Ew+)   (2) (0A0+) + (0B0+) + (0Ew+)
(3) (0Ev+) + (0Aq+) + (0Bq+)   (4) (0A0+) + (0B0+) + (0Aq+) + (0Bq+)

(0Em−) × (0A0±) = (0Em−) × (0B0±) = (0Em∓)
(0Em−) × (0Em′±) = (1) (0Ev∓) + (0Ew∓)   (2) (0A0∓) + (0B0∓) + (0Ew∓)
(3) (0Ev∓) + (0Aq∓) + (0Bq∓)   (4) (0A0∓) + (0B0∓) + (0Aq∓) + (0Bq∓)

(kEA0) × (0A0±) = (kEA0)   (kEA0) × (0B0±) = (kEB0)   (kEA0) × (0Em±) = (kGm)
(kEA0) × (k′EA0) = (a) (xEA0) + (yEA0)   (b) (0A0+) + (0A0−) + (yEA0)
(c) (xEA0) + (πA0+) + (πA0−)   (d) (0A0+) + (0A0−) + (πA0+) + (πA0−)

(kEB0) × (0A0±) = (kEB0)   (kEB0) × (0B0±) = (kEA0)   (kEB0) × (0Em±) = (kGm)
(kEB0) × (k′EA0) = (a) (xEB0) + (yEB0)   (b) (0B0+) + (0B0−) + (yEB0)
(c) (xEB0) + (πB0+) + (πB0−)   (d) (0B0+) + (0B0−) + (πB0+) + (πB0−)
(kEB0) × (k′EB0) = (a) (xEA0) + (yEA0)   (b) (0A0+) + (0A0−) + (yEA0)
(c) (xEA0) + (πA0+) + (πA0−)   (d) (0A0+) + (0A0−) + (πA0+) + (πA0−)

(kGm) × (0A0±) = (kGm) × (0B0±) = (kGm)
(kGm) × (0Em′±) = (1) (kGv) + (kGw)   (2) (kEA0) + (kEB0) + (kGw)
(3) (kGv) + (kEAq) + (kEBq)   (4) (kEA0) + (kEB0) + (kEAq) + (kEBq)
(kGm) × (k′EA0) = (kGm) × (k′EB0) =
(a) (xGm) + (yGm)   (b) (0Em+) + (0Em−) + (yGm)
(c) (xGm) + (πEm+) + (πEm−)   (d) (0Em+) + (0Em−) + (πEm+) + (πEm−)

(kGm) × (k′Gm′) =
(a1) (xGv) + (xGw) + (yGv) + (yGw)
(a2) (xEA0) + (xEB0) + (xGw) + (yEA0) + (yEB0) + (yGw)
(a3) (xGv) + (xEAq) + (xEBq) + (yGv) + (yEAq) + (yEBq)
(a4) (xEA0) + (xEB0) + (xEAq) + (xEBq) + (yEA0) + (yEB0) + (yEAq) + (yEBq)
(b1) (0Ev+) + (0Ev−) + (0Ew+) + (0Ew−) + (yGv) + (yGw)
(b2) (0A0+) + (0A0−) + (0B0+) + (0B0−) + (0Ew+) + (0Ew−) + (yEA0) + (yEB0) + (yGw)
(b3) (0Ev+) + (0Ev−) + (0Aq+) + (0Aq−) + (0Bq+) + (0Bq−) + (yGv) + (yEAq) + (yEBq)
(b4) (0A0+) + (0A0−) + (0B0+) + (0B0−) + (0Aq+) + (0Aq−) + (0Bq+) + (0Bq−) + (yEA0) + (yEB0) + (yEAq) + (yEBq)

Table 2. (continued)

(c1) $(xGv)+(xGw)+(\pi Ev+)+(\pi Ev-)+(\pi Ew+)+(\pi Ew-)$
(c2) $(xEA0)+(xEB0)+(xGw)+(\pi A0+)+(\pi A0-)+(\pi B0+)+(\pi B0-)+(\pi Ew+)+(\pi Ew-)$
(c3) $(xGv)+(xEAq)+(xEBq)+(\pi Ev+)+(\pi Ev-)+(\pi Aq+)+(\pi Aq-)+(\pi Bq+)+(\pi Bq-)$
(c4) $(xEA0)+(xEB0)+(xEAq)+(xEBq)+(\pi A0+)+(\pi A0-)+(\pi B0+)+(\pi B0-)+(\pi Aq+)+(\pi Aq-)$ $+(\pi Bq+)+(\pi Bq-)$
(d1) $(0Ev+)+(0Ev-)+(0Ew+)+(0Ew-)+(\pi Ev+)+(\pi Ev-)+(\pi Ew+)+(\pi Ew-)$
(d2) $(0A0+)+(0A0-)+(0B0+)+(0B0-)+(0Ew+)+(0Ew-)+(\pi A0+)+(\pi A0-)+(\pi B0+)+(\pi B0-)$ $+(\pi Ew+)+(\pi Ew-)$
(d3) $(0Ev+)+(0Ev-)+(0Aq+)+(0Aq-)+(0Bq+)+(0Bq-)+(\pi Ev+)+(\pi Ev-)+(\pi Aq+)+(\pi Aq-)$ $+(\pi Bq+)+(\pi Bq-)$
(d4) $(0A0+)+(0A0-)+(0B0+)+(0B0-)+(0Aq+)+(0Aq-)+(0Bq+)+(0Bq-)+(\pi A0+)+(\pi A0-)$ $+(\pi B0)+(\pi B0-)+(\pi Aq+)+(\pi Aq-)+(\pi Bq+)+(\pi Bq-)$

$(\boldsymbol{\pi A0+})\times(0A0\pm)=(\pi A0\pm)$	$(\pi A0+)\times(0B0\pm)=(\pi B0\pm)$	$(\pi A0+)\times(0Em\pm)=(\pi Em\pm)$
$(\pi A0+)\times(kEA0)=(zEA0)$	$(\pi A0+)\times(kEB0)=(zEB0)$	$(\pi A0+)\times(kGm)=(zGm)$
$(\pi A0+)\times(\pi A0+)=(0A0+)$		
$(\boldsymbol{\pi A0-})\times(0A0\pm)=(\pi A0\mp)$	$(\pi A0-)\times(0B0\pm)=(\pi B0\mp)$	$(\pi A0-)\times(0Em\pm)=(\pi Em\mp)$
$(\pi A0-)\times(kEA0)=(zEA0)$	$(\pi A0-)\times(kEB0)=(zEB0)$	$(\pi A0-)\times(kGm)=(zGm)$
$(\pi A0-)\times(\pi A0\pm)=(0A0\mp)$		
$(\boldsymbol{\pi B0+})\times(0A0\pm)=(\pi B0\pm)$	$(\pi B0+)\times(0B0\pm)=(\pi A0\pm)$	$(\pi B0+)\times(0Em\pm)=(\pi Em\pm)$
$(\pi B0+)\times(kEA0)=(zEB0)$	$(\pi B0+)\times(kEB0)=(zEA0)$	$(\pi B0+)\times(kGm)=(zGm)$
$(\pi B0+)\times(\pi A0\pm)=(0B0\pm)$	$(\pi B0+)\times(\pi B0+)=(0A0+)$	
$(\boldsymbol{\pi B0-})\times(0A0\pm)=(\pi B0\mp)$	$(\pi B0-)\times(0B0\pm)=(\pi A0\mp)$	$(\pi B0-)\times(0Em\pm)=(\pi Em\mp)$
$(\pi B0-)\times(kEA0)=(zEB0)$	$(\pi B0-)\times(kEB0)=(zEA0)$	$(\pi B0-)\times(kGm)=(zGm)$
$(\pi B0-)\times(\pi A0\pm)=(0B0\mp)$	$(\pi B0-)\times(\pi B0\pm)=(0A0\mp)$	

$(\boldsymbol{\pi Em+})\times(0A0\pm)=$	$(\pi Em+)\times(0B0\pm)=(\pi Em\pm)$	
$(\pi Em+)\times(0\dot{E}m'\pm)=$	(1) $(\pi Ev\pm)+(\pi Ew\pm)$	(2) $(\pi A0\pm)+(\pi B0\pm)+(\pi Ew\pm)$
	(3) $(\pi Ev\pm)+(\pi Aq\pm)+(\pi Bq\pm)$	(4) $(\pi A0\pm)+(\pi B0\pm)+(\pi Aq\pm)+(\pi Bq\pm)$
$(\pi Em+)\times(kEA0)=$	$(\pi Em+)\times(kEb0)=(zGm)$	
$(\pi Em+)\times(kGm')=$	(1) $(zGv)+(zGw)$	(2) $(zEA0)+(zEB0)+(zGw)$
	(3) $(zGv)+(zEAq)+(zEBq)$	(4) $(zEA0)+(zEB0)+(zEAq)+(zEBq)$
$(\pi Em+)\times(\pi A0\pm)=$	$(\pi Em+)\times(\pi B0\pm)=(0Em\pm)$	
$(\pi Em+)\times(\pi Em'+)=$	(1) $(0Ev+)+(0Ew+)$	(2) $(0A0+)+(0B0+)+(0Ew+)$
	(3) $(0Ev+)+(0Aq+)+(0Bq+)$	(4) $(0A0+)+(0B0+)+(0Aq+)+(0Bq+)$
$(\boldsymbol{\pi Em-})\times(0A0\pm)=$	$(\pi Em-)\times(0B0\pm)=(\pi Em\mp)$	
$(\pi Em-)\times(0Em'\pm)=$	(1) $(\pi Ev\mp)+(\pi Ew\mp)$	(2) $(\pi A0\mp)+(\pi B0\mp)+(\pi Ew\mp)$
	(3) $(\pi Ev\mp)+(\pi Aq\mp)+(\pi Bq\mp)$	(4) $(\pi A0\mp)+(\pi B0\mp)+(\pi Aq\mp)+(\pi Bq\mp)$
$(\pi Em-)\times(kEA0)=$	$(\pi Em-)\times(kEB0)=(zGm)$	
$(\pi Em-)\times(kGm')=$	(1) $(zGv)+(zGw)$	(2) $(zEA0)+(zEB0)+(zGw)$
	(3) $(zGv)+(zEAq)+(zEBq)$	(4) $(zEA0)+(zEB0)+(zEAq)+(zEBq)$
$(\pi Em-)\times(\pi A0\pm)=$	$(\pi Em-)\times(\pi B0\pm)=(0Em\mp)$	
$(\pi Em-)\times(\pi Em'\pm)=$	(1) $(0Ev\mp)+(0Ew\mp)$	(2) $(0A0\mp)+(0B0\mp)+(0Ew\mp)$
	(3) $(0Ev\mp)+(0Aq\mp)+(0Bq\mp)$	(4) $(0A0\mp)+(0B0\mp)+(0Aq\mp)+(0Bq\mp)$

and only for $n=2q=2,4,6,\ldots$

$(\boldsymbol{0Aq+})\times(0A0\pm)=(0Aq\pm)$	$(0Aq+)\times(0B0\pm)=(0Bq\pm)$	$(0Aq+)\times(0Em\pm)=(0Eu\pm)$
$(0Aq+)\times(kEA0)=(kEAq)$	$(0Aq+)\times(kEB0)=(kEBq)$	$(0Aq+)\times(kGm)=(kGu)$
$(0Aq+)\times(\pi A0\pm)=(\pi Aq\pm)$	$(0Aq+)\times(\pi B0\pm)=(\pi Bq\pm)$	$(0Aq+)\times(\pi Em\pm)=(\pi Eu\pm)$
$(0Aq+)\times(0Aq+)=(0A0+)$		
$(\boldsymbol{0Aq-})\times(0A0\pm)=(0Aq\mp)$	$(0Aq-)\times(0B0\pm)=(0Bq\mp)$	$(0Aq-)\times(0Em\pm)=(0Eu\mp)$
$(0Aq-)\times(kEA0)=(kEAq)$	$(0Aq-)\times(kEB0)=(kEBq)$	$(0Aq-)\times(kGm)=(kGu)$
$(0Aq-)\times(\pi A0\pm)=(\pi Aq\mp)$	$(0Aq-)\times(\pi B0\pm)=(\pi Bq\mp)$	$(0Aq-)\times(\pi Em\pm)=(\pi Eu\mp)$
$(0Aq-)\times(0Aq\pm)=(0A0\mp)$		

Table 2. (continued)

(0Bq+)×(0A0±) = (0Bq±)	(0Bq+)×(0B0±) = (0Aq±)	(0Bq+)×(0Em±) = (0Eu±)
(0Bq+)×(kEA0) = (kEBq)	(0Bq+)×(kEB0) = (kEAq)	(0Bq+)×(kGm) = (kGu)
(0Bq+)×(πA0±) = (πBq±)	(0Bq+)×(πB0±) = (πAq±)	(0Bq+)×(πEm±) = (πEu±)
(0Bq+)×(0Aq±) = (0B0±)	(0Bq+)×(0Bq+) = (0A0+)	
(0Bq−)×(0A0±) = (0Bq∓)	(0Bq−)×(0B0±) = (0Aq∓)	(0Bq−)×(0Em±) = (0Eu∓)
(0Bq−)×(kEA0) = (kEBq)	(0Bq−)×(kEB0) = (kEAq)	(0Bq−)×(kGm) = (kGu)
(0Bq−)×(πA0±) = (πBq∓)	(0Bq−)×(πB0±) = (πAq∓)	(0Bq−)×(πEm±) = (πEu∓)
(0Bq−)×(0Aq±) = (0B0∓)	(0Bq−)×(0Bq±) = (0A0∓)	
(kEAq)×(0A0±) = (kEAq)	(kEAq)×(0B0±) = kEBq)	(kEAq)×(0Em±) = (kGu)
(kEAq)×(k′(EA0) =	(a) (xEAq)+(yEAq)	(b) (0Aq+)+(0Aq−)+(yEAq)
	(c) (xEAq)+(πAq+)(+πAq−)	(d) (0Aq+)+(0Aq−)+(πAq+)+(πAq−)
(kEAq)×(k′EB0) =	(a) (xEBq)+(yEBq)	(b) (0Bq+)+(0Bq−)+(yEBq)
	(c) (xEBq)+(πBq+)+(πBq−)	(d) (0Bq+)+(0Bq−)+(πBq+)+(πBq−)
(kEAq)×(k′Gm) =	(a) (xGu)+(yGu)	(b) (0Eu+)+(0Eu−)+(yGu)
	(c) (xGu)+(πEu+)+(πEu−)	(d) (0Eu+)+(0Eu−)+(πEu+)+(πEu−)
(kEAq)×(πA0±) = (zEAq)	(kEAq)×(πB0±) = (zEBq)	(kEAq)×(πEm±) = (zGu)
(kEAq)×(0Aq±) = (kEA0)	(kEAq)×(0Bq±) = (kEB0)	
(kEAq)×(k′EAq) =	(a) (xEA0)+(yEA0)	(b) (0A0+)+(0A0−)+(yEA0)
	(c) (xEA0)+(πA0+)+(πA0−)	(d) (0A0+)+(0A0−)+(πA0+)+(πA0−)
(kEBq)×(0A0±) = (kEBq)	(kEBq)×(0B0±) = (kEAq)	(kEBq)×(0Em±) = (kGu)
(kEBq)×(k′EA0) =	(a) (xEBq)+(yEBq)	(b) (0Bq+)+(0Bq−)+(yEBq)
	(c) (xEBq)+(πBq+)+(πBq−)	(d) (0Bq+)+(0Bq−)+(πBq+)+(πBq−)
(kEBq)×(k′EB0) =	(a) (xEAq)+(yEAq)	(b) (0Aq+)+(0Aq−)+(yEAq)
	(c) (xEAq)+(πAq+)+(πAq−)	(d) (0Aq+)+(0Aq−)+(πAq+)+(πAq−)
(kEBq)×(k′Gm) =	(a) (xGu)+(yGu)	(b) (0Eu+)+(0Eu−)+(yGu)
	(c) (xGu)+(πEu+)+(πEu−)	(d) (0Eu+)+(0Eu−)+(πEu+)+(πEu−)
(kEBq)×(πA0±) = (zEBq)	(kEBq)×(πB0±) = (zEAq)	(kEBq)×(πEm±) = (zGu)
(kEBq)×(0Aq±) = (kEB0)	(kEBq)×(0Bq±) = (kEA0)	
(kEBq)×(k′EAq) =	(a) (xEB0)+(yEB0)	(b) (0B0+)+(0B0−)+(yEB0)
	(c) (xEA0)+(πB0+)+(πB0−)	(d) (0B0+)+(0B0−)+(πB0+)+(πB0−)
(kEBq)×(k′EBq) =	(a) (xEA0)+(yEA0)	(b) (0A0+)+(0A0−)+(yEA0)
	(c) (xEA0)+(πA0+)+(πA0−)	(d) (0A0+)+(0A0−)+(πA0+)+(πA0−)
(πAq+)×(0A0±) = (πAq±)	(πAq+)×(0B0±) = (πBq±)	(πAq+)×(0Em±) = (πEu±)
(πAq+)×(kEA0) = (zEAq)	(πAq+)×(kEB0) = (zEBq)	(πAq+)×(kGm) = (zGu)
(πAq+)×(πA0±) = (0Aq±)	(πAq±)×(πB0±) = (0Bq±)	(πAq+)×(πEm±) = (0Eu±)
(πAq+)×(0Aq±) = (πA0±)	(πAq+)×(0Bq±) = (πB0±)	(πAq+)×(kEAq) = (zEA0)
(πAq+)×(kEBq) = (zEB0)	(πAq+)×(πAq+) = (0A0+)	
(πAq−)×(0A0±) = (πAq∓)	(πAq−)×(0B0±) = (πBq∓)	(πAq−)×(0Em±) = (πEu∓)
(πAq−)×(kEA0) = (zEAq)	(πAq−)×(kEB0) = (zEBq)	(πAq−)×(kGm) = (zGu)
(πAq−)×(πA0±) = (0Aq∓)	(πAq−)×(πB0±) = (0Bq∓)	(πAq−)×(πEm±) = (0Eu∓)
(πAq−)×(0Aq±) = (πA0∓)	(πAq−)×(0Bq±) = (πB0∓)	(πAq−)×(kAEq) = (zEA0)
(πAq−)×(kEBq) = (zEB0)	(πAq−)×(πAq±) = (0A0∓)	
(πBq+)×(0A0±) = (πBq±)	(πBq+)×(0B0±) = (πAq±)	(πBq+)×(0Em±) = (πEu±)
(πBq+)×(kEA0) = (zEBq)	(πBq+)×(kEB0) = (zEAq)	(πBq+)×(kGm) = (zGu)
(πBq+)×(πA0±) = (0Bq±)	(πBq+)×(πB0±) = (0Aq±)	(πBq+)×(Em±) = (0Eu±)
(πBq+)×(0Aq±) = (πB0±)	(πBq+)×(0Bq±) = (πA0±)	(πBq+)×(kEAq) = (zEB0)
(πBq+)×(kEBq) = (zEA0)	(πBq+)×(πAq±) = (0B0±)	(πBq+)×(πBq+) = (0A0+)
(πBq−)×(0A0±) = (πBq∓)	(πBq−)×(0B0±) = (πAq∓)	(πBq−)×(0Em±) = (πEu∓)
(πBq−)×(kEA0) = (zEBq)	(πBq−)×(kEB0) = (zEAq)	(πBq−)×(kGm) = (zGu)
(πBq−)×(πA0±) = (0Bq∓)	(πBq−)×(πB0±) = (0Aq∓)	(πBq−)×(πEm±) = (0Eu∓)
(πBq−)×(0Aq±) = (πB0∓)	(πBq−)×(0Bq±) = (πA0∓)	(πBq−)×(kEAq) = (zEB0)
(πBq−)×(kEBq) = (zEA0)	(πBq−)×(πAq±) = (0B0∓)	(πBq−)×(πBq±) = (0A0∓)

3.2. *The non-symmorphic line groups* $L(\overline{2n})2c$ $(n = 1, 3, \ldots)$ *and* Ln/mcc $(n = 2, 4, \ldots)$

Table 3. The characters of the reps of the line groups $L(\overline{2n})2c$ (n odd) and Ln/mcc (n even). For s, t, α, k, m and q see the caption of table 1.

Rep	$(C_n^s \mid t)$	$(\sigma_v C_n^s \mid \frac{1}{2}+t)$	$(\sigma_h C_n^s \mid -t)$	$(\sigma_h \sigma_v C_n^s \mid -\frac{1}{2}+t)$
$(0A0\pm) = {}_0A_0^{\mp}$	1	1	± 1	± 1
$(0B0\pm) = {}_0B_0^{\pm}$	1	-1	± 1	∓ 1
$(0Em\pm) = {}_0E_{m,-m}^{\pm}$	$2\cos ms\alpha$	0	$\pm 2\cos ms\alpha$	0
$(kEA0) = {}_k^{-k}E_{A_0}$	$2\cos kt$	$2\cos k(\frac{1}{2}+t)$	0	0
$(kEB0) = {}_k^{-k}E_{B_0}$	$2\cos kt$	$-2\cos k(\frac{1}{2}+t)$	0	0
$(kGm) = {}_k^{-k}G_{m,-m}$	$4\cos kt \cos ms\alpha$	0	0	0
$(\pi E0) = {}_\pi E_0$	$2(-1)^t$	0	0	0
$(\pi Em\pm) = {}_\pi E_{m,-m}^{\pm}$	$(-1)^t 2\cos ms\alpha$	0	$\pm(-1)^t 2\mathrm{i}\sin ms\alpha$	0
and only for $n = 2q = 2, 4, 6, \ldots$				
$(0Aq\pm) = {}_0A_q^{\pm}$	$(-1)^s$	$(-1)^s$	$\pm(-1)^s$	$\pm(-1)^s$
$(0Bq\pm) = {}_0B_{q\pm}^{\pm}$	$(-1)^s$	$-(-1)^s$	$\pm(-1)^s$	$\mp(-1)^s$
$(kEAq) = {}_k^{-k}E_{A_q}$	$(-1)^s 2\cos kt$	$(-1)^s 2\cos k(\frac{1}{2}+t)$	0	0
$(kEBq) = {}_k^{-k}E_{B_q}$	$(-1)^s 2\cos kt$	$-(-1)^s 2\cos k(\frac{1}{2}+t)$	0	0
$(\pi Eq) = {}_\pi E_q$	$2(-1)^{s+t}$	0	0	0

Table 4. Decompositions of the Kronecker products of reps of $L(\overline{2n})2c$ (n odd) and Ln/mcc (n even). The rep symbols are defined in table 3.

$\mathbf{(0A0+)} \times (0A0+) = (0A0+)$		
$\mathbf{(0A0-)} \times (0A0\pm) = (0A0\mp)$		
$\mathbf{(0B0+)} \times (0A0\pm) = (0B0\pm)$	$(0B0+) \times (0B0+) = (0A0+)$	
$\mathbf{(0B0-)} \times (0A0\pm) = (0B0\mp)$	$(0B0-) \times (0B0\pm) = (0A0\mp)$	
$\mathbf{(0Em+)} \times (0A0\pm) = (0Em+) \times (0B0\pm) = (0Em\pm)$		
$(0Em+) \times (0Em'+) =$	(1) $(0Ev+) + (0Ew+)$	(2) $(0A0+) + (0B0+) + (0Ew+)$
	(3) $(0Ev+) + (0Aq+) + (0Bq+)$	(4) $(0A0+) + (0B0+) + (0Aq+) + (0Bq+)$
$\mathbf{(0Em-)} \times (0A0\pm) = (0Em-) \times (0B0\pm) = (0Em\mp)$		
$(0Em-) \times (0Em'\pm) =$	(1) $(0Ev\mp) + (0Ew\mp)$	(2) $(0A0\mp) + (0B0\mp) + (0Ew\mp)$
	(3) $(0Ev\mp) + (0Aq\mp) + (0Bq\mp)$	(4) $(0A0\mp) + (0B0\mp) + (0Aq\mp) + (0Bq\mp)$
$\mathbf{(kEA0)} \times (0A0\pm) = (kEA0)$	$(kEA0) \times (0B0\pm) = (kEB0)$	$(kEA0) \times (0Em\pm) = (kGm)$
$(kEA0) \times (k'EA0) =$	(e) $(xEA0) + (yEA0)$	(f) $(xEA0) + (yEB0)$
	(g) $(0A0+) + (0A0-) + (yEA0)$	(h) $(0A0+) + (0A0-) + (yEB0)$
	(c) $(xEA0) + (\pi E0)$	(d) $(0A0+) + (0A0-) + (\pi E0)$
$\mathbf{(kEB0)} \times (0A0\pm) = (kEB0)$	$(kEB0) \times (0B0\pm) = (kEA0)$	$(kEB0) \times (0Em\pm) = (kGm)$
$(kEB0) \times (k'EA0) =$	(e) $(xEB0) + (yEB0)$	(f) $(xEB0) + (yEA0)$
	(g) $(0B0+) + (0B0-) + (yEB0)$	(h) $(0B0+) + (0B0-) + (yEA0)$
	(c) $(xEB0) + (\pi E0)$	(d) $(0B0+) + (0B0-) + (\pi E0)$
$(kEB0) \times (k'EB0) =$	(e) $(xEA0) + (yEA0)$	(f) $(xEA0) + (yEB0)$
	(g) $(0A0+) + (0A0-) + (yEA0)$	(h) $(0A0+) + (0A0-) + (yEB0)$
	(c) $(xEA0) + (\pi E0)$	(d) $(0A0+) + (0A0-) + (\pi E0)$
$\mathbf{(kGm)} \times (0A0\pm) = (kGm) \times (0B0\pm) = (kGm)$		
$(kGm) \times (0Em'\pm) =$	(1) $(kGv) + (kGw)$	(2) $(kEA0) + (kEB0) + (kGw)$
	(3) $(kGv) + (kEAq) + (kEBq)$	(4) $(kEA0) + (kEB0) + (kEAq) + (kEBq)$
$(kGm) = (k'EA0) = (kGm) \times (k'EB0) =$		
	(a) $(xGm) + (yGm)$	(b) $(0Em+) + (0Em-) + (yGm)$
	(c) $(xGm) + (\pi Em+) + (\pi Em-)$	(d) $(0Em+) + (0Em-) + (\pi Em+) + (\pi Em-)$

Table 4. (continued)

(kGm) × (k′Gm′) =
(a1) (xGv) + (xGw) + (yGv) + (yGw)
(a2) (xEA0) + (xEB0) + (xGv) + (yEA0) + (yEB0) + (yGv)
(a3) (xGv) + (xEAq) + (xEBq) + (yGv) + (yEAq) + (yEBq)
(a4) (xEA0) + (xEB0) + (xEAq) + (xEBq) + (yEA0) + (yEB0) + (yEAq) + (yEBq)
(b1) (0Ev+) + (0Ev−) + (0Ew+) + (0Ew−) + (yGv) + (yGw)
(b2) (0A0) + (0A0−) + (0B0+) + (0B0−) + (0Ew+) + (0Ew−) + (yEA0) + (yEB0) + (yGw)
(b3) (0Ev+) + (0Ev−) + (0Aq+) + (0Aq−) + (0Bq+) + (0Bq−) + (yGv) + (yEAq) + (yEBq)
(b4) (0A0+) + (0A0−) + (0B0+) + (0B0−) + (0Aq+) + (0Aq−) + (0Bq+) + (0Bq−) + (yEA0) + (yEB0) + (yEAq) + (yEBq)
(c1) (xGv) + (xGw) + (πEv+) + (πEv−) + (πEw+) + (πEw−)
(c2) (xEA0) + (xEB0) + (xGw) + 2(πE0) + (πEw+) + (πEw−)
(c3) (xGv) + (xEAq) + (xEBq) + (πEv+) + (πEv−) + 2(πEq)
(c4) (xEA0) + (xEB0) + (xEAq) + (xEBq) + 2(πE0) + 2(πEq)
(d1) (0Ev+) + (0Ev−) + (0Ew+) + (0Ew−) + (πEv+) + (πEv−) + (πEw+) + (πEw−)
(d2) (0A0+) + (0A0−) + (0B0+) + (0B0−) + (0Ew+) + (0Ew−) + 2(πE0) + (πEw−) − (πEw−)
(d3) (0Ev+) + (0Ev−) + (0Aq+) + (0Aq−) + (0Bq+) + (0Bq−) + (πEv+) + (πEv−) + 2(πEq)
(d4) (0A0+) + (0A0−) + (0B0+) + (0B0−) + (0Aq+) + (0Aq−) + (0Bq+) + (0Bq−) + 2(πE0) + 2(πEq)

(πE0) × (0A0±) = (πE0) × (0B0±) = (πE0)   (πE0) × (0Em±) = (πEm+) + (πEm−)
(πE0) × (kEA0) = (πE0) × (kEB0) = (zEA0) + (zEB0)   (πE0) × (kGm) = 2(zGm)
(πE0) × (πE0) = (0A0+) + (0A0−) + (0B0+) + (0B0−)

(πEm+) × (0A0±) = (πEm+) × (0B0±) = (πEm±)

(πEm+) × (0Em′±) =	(1) (πEv±) + (πEw±)	(2) (πE0) + (πEw±)
	(3) (πEv±) + (πEq)	(4) (πE0) + (πEq)
(πEm+) × (kEA0) =	(πEm+) × (kEB0) = (zGm)	
(πEm+) × (kGm′) =	(1) (zGv) + (zGw)	(2) (zEA0) + (zEB0) + (zGw)
	(3) (zGv) + (zEAq) + (zEBq)	(4) (zEA0) + (zEB0) + (zEAq) + (zEBq)
(πEm+) × (πE0) = (0Em+) + (0Em−)		
(πEm+) × (πEm′+) =	(1) (0Ev−) + (0Ew+)	(2) (0A0−) + (0B0−) + (0Ew+)
	(3) (0Ev−) + (0Aq+) + (0Bq+)	(4) (0A0−) + (0B0−) + (0Aq+) + (0Bq+)

(πEm−) × (0A0±) = (πEm−) × (0B0±) = (πEm∓)

(πEm−) × (0Em′±) =	(1) (πEv∓) + (πEw∓)	(2) (πE0) + (πEw∓)
	(3) (πEv∓) + (πEq)	(4) (πE0) + (πEq)
(πEm−) × (kEA0) =	(πEm−) × (kEB0) = (zGm)	
(πEm−) × (kGm′) =	(1) (zGv) + (zGw)	(2) (zEA0) + (zEB0) + (zGw)
	(3) (zGv) + (zEAq) + (zEBq)	(4) (zEA0) + (zEB0) + (zEAq) + (zEBq)
(πEm−) × (πE0) = (0Em+) + (0Em−)		
(πEm−) × (πEm′±) =	(1) (0Ev±) + (0Ew±)	(2) (0A0±) + (0B0±) + (0Ew∓)
	(3) (0Ev±) + (0Aq∓) + (0Bq∓)	(4) (0A0±) + (0B0±) + (0Aq∓) + (0Bq∓)

and only for $n = 2, 4, 6, \ldots$

(0Aq+) × (0A0±) = (0Aq±)	(0Aq+) × (0B0±) = (0Bq±)	(0Aq+) × (0Em±) = (0Eu±)
(0Aq+) × (kEA0) = (kEAq)	(0Aq+) × (kEB0) = (kEBq)	(0Aq+) × (kGm) = (kGu)
(0Aq+) × (πE0) = (πEq)	(0Aq+) × (πEm±) + (πEu∓)	(0Aq+) × (0Aq+) = (0A0+)
(0Aq−) × (0A0±) = (0Aq∓)	(0Aq−) × (0B0±) = (0Bq∓)	(0Aq−) × (0Em±) = (0Eu∓)
(0Aq−) × (kEA0) = (kEAq)	(0Aq−) × (kEB0) = (kEBq)	(0Aq−) × (kGm) = (kGu)
(0Aq−) × (πE0) = (πEq)	(0Aq−) × (πEm±) = (πEu±)	(0Aq−) × (0Aq±) = (0A0∓)
(0Bq+) × (0A0±) = (0Bq±)	(0Bq+) × (0B0±) = (0Aq±)	(0Bq+) × (0Em±) = (0Eu±)
(0Bq+) × (kEA0) = (kEBq)	(0Bq+) × (kEB0) = (kEAq)	(0Bq+) × (kGm) = (kGu)
(0Bq+) × (πE0) = (πEq)	(0Bq+) × (πEm±) = (πEu∓)	(0Bq+) × (0Aq±) = (0B0±)
(0Bq+) × (0Bq+) = (0A0+)		

Table 4. (continued)

$(0Bq-)\times(0A0\pm)=(0Bq\mp)$ $\quad(0Bq-)\times(0B0\pm)=(0Aq\mp)$ $\quad(0Bq-)\times(0Em\pm)=(0Eu\mp)$
$(0Bq-)\times(kEA0)=(kEBq)$ $\quad(0Bq-)\times(kEB0)=(kEAq)$ $\quad(0Bq-)\times(kGm)=(kGu)$
$(0Bq-)\times(\pi E0)=(\pi Eq)$ $\quad(0Bq-)\times(\pi Em\pm)=(\pi Eu\pm)$ $\quad(0Bq-)\times(0Aq\pm)=(0B0\mp)$
$(0Bq-)\times(0Bq\pm)=(0A0\mp)$

$(kEAq)\times(0A0\pm)=(kEAq)$ $\quad(kEAq)\times(0B0\pm)=(kEBq)$ $\quad(kEAq)\times(0Em\pm)=(kGu)$
$(kEAq)\times(k'EA0)=$ (e) $(xEAq)+(yEAq)$ (f) $(xEAq)+(yEBq)$
(g) $(0Aq+)+(0Aq-)+(yEAq)$ (h) $(0Aq+)+(0Aq-)+(yEBq)$
(c) $(xEAq)+(\pi Eq)$ (d) $(0Aq+)+(0Aq-)+(\pi Eq)$
$(kEAq)\times(k'EB0)=$ (e) $(xEBq)+(yEBq)$ (f) $(xEBq)+(yEAq)$
(g) $(0Bq+)+(0Bq-)+(yEBq)$ (h) $(0Bq+)+(0Bq-)+(yEAq)$
(c) $(xEBq)+(\pi Eq)$ (d) $(0Bq+)+(0Bq-)+(\pi Eq)$
$(kEAq)\times(k'Gm)=$ (a) $(xGu)+(yGu)$ (b) $(0Eu+)+(0Eu-)+(yGu)$
(c) $(xGu)+(\pi Eu+)+(\pi Eu-)$ (d) $(0Eu+)+(0Eu-)+(\pi Eu+)+(\pi Eu-)$
$(kEAq)\times(\pi E0)=(zEAq)+(zEBq)$ $\quad(kEAq)\times(\pi Em\pm)=(zGu)$
$(kEAq)\times(0Aq\pm)=(kEA0)$ $\quad(kEAq)\times(0Bq\pm)+(kEB0)$
$(kEAq)\times(k'EAq)=$ (e) $(xEA0)+(yEA0)$ (f) $(xEA0)+(yEB0)$
(g) $(0A0+)+(0A0-)+(yEA0)$ (h) $(0A0+)+(0A0-)+(yEB0)$
(c) $(xEA0)+(\pi E0)$ (d) $(0A0+)+(0A0-)+(\pi E0)$

$(kEBq)\times(0A0)=(kEBq)$ $\quad(kEBq)\times(0B0\pm)=(kEAq)$ $\quad(kEBq)\times(0Em\pm)=(kGu)$
$(kEBq)\times(k'EA0)=$ (e) $(xEBq)+(yEBq)$ (f) $(xEBq)+(yEAq)$
(g) $(0Bq+)+(0Bq-)+(yEBq)$ (h) $(0Bq+)+(0Bq-)+(yEAq)$
(c) $(xEBq)+(\pi Eq)$ (d) $(0Bq+)+(0Bq-)+(\pi Eq)$
$(kEBq)\times(k'EB0)=$ (e) $(xEAq)+(yEAq)$ (f) $(xEAq)+(yEBq)$
(g) $(0Aq+)+(0Aq-)+(yEAq)$ (h) $(0Aq+)+(0Aq-)+(yEBq)$
(c) $(xEAq)+(\pi Eq)$ (d) $(0Aq+)+(0Aq-)+(\pi Eq)$
$(kEBq)\times(k'Gm)=$ (a) $(xGu)+(yGu)$ (b) $(0Eu+)+(0Eu-)+(yGu)$
(c) $(xGu)+(\pi Eu+)+(\pi Eu-)$ (d) $(0Eu+)+(0Eu-)+(\pi Eu+)+(\pi Eu-)$
$(kEBq)\times(\pi E0)=(zEAq)+(zEBq)$ $\quad(kEBq)\times(\pi Em\pm)=(zGu)$
$(kEBq)\times(0Aq\pm)=(kEB0)$ $\quad(kEBq)=(0Bq\pm)=(kEA0)$
$(kEBq)\times(k'EAq)=$ (e) $(xEB0)+(yEB0)$ (f) $(xEB0)+(yEA0)$
(g) $(0B0+)+(0B0-)+(yEB0)$ (h) $(0B0+)+(0B0-)+(yEA0)$
(c) $(xEB0)+(\pi E0)$ (d) $(0B0+)+(0B0-)+(\pi E0)$
$(kEBq)\times(k'EBq)=$ (e) $(xEA0)+(yEA0)$ (f) $(xEA0)+(yEB0)$
(g) $(0A0+)+(0A0-)+(yEA0)$ (h) $(0A0+)+(0A0-)+(yEB0)$
(c) $(xEA0)+(\pi E0)$ (d) $(0A0+)+(0A0-)+(\pi E0)$

$(\pi Eq)\times(0A0\pm)=(\pi Eq)\times(0B0\pm)=(\pi Eq)$ $\quad(\pi Eq)\times(0Em\pm)=(\pi Eu+)+(\pi Eu-)$
$(\pi Eq)\times(kEA0)=(\pi Eq)\times(kEB0)=(zEAq)+(zEBq)$ $\quad(\pi Eq)\times(kGm)=2(zGu)$
$(\pi Eq)\times(\pi E0)=(0Aq+)+(0Aq-)+(0Bq+)+(0Bq-)$ $\quad(\pi Eq)\times(\pi Em\pm)=(0Eu+)+(0Eu-)$
$(\pi Eq)\times(0Aq\pm)=(\pi Eq)\times(0Bq\pm)=(\pi E0)$
$(\pi Eq)\times(kEAq)=(\pi Eq)\times(kEBq)=(zEA0)+(zEB0)$
$(\pi Eq)\times(\pi Eq)=(0A0+)+(0A0-)+(0B0+)+(0B0-)$

3.3. *The non-symmorphic line groups $L(2q)_q/mcm$ $(n=2q=2,4,\ldots)$*

Table 5. The characters of the reps of the line groups $L(2q)_q/mcm$. For q, α, t, m and k see the caption of table 1; here $r=0,1,\ldots,q-1$, $j=1,2,\ldots,(q-1)/2$ for q odd and $j=1,2,\ldots,(q-2)/2$ for q even; $\bar{j}=q-j$; $h=q/2$ for q even.

Rep	$(C_{2q}^{2r}\|t)$	$(C_{2q}^{2r+1}\|\frac{1}{2}+t)$	$(\sigma_v C_{2q}^{2r}\|t)$	$(\sigma_v C_{2q}^{2r+1}\|\frac{1}{2}+t)$	$(\sigma_h C_{2q}^{2r}\|-t)$	$(\sigma_h C_{2q}^{2r+1}\|-\frac{1}{2}-t)$	$(\sigma_h\sigma_v C_{2q}^{2r}\|-t)$	$(\sigma_h\sigma_v C_{2q}^{2r+1}\|-\frac{1}{2}-t)$
$(0A0\pm)={}_0A_0^{\pm}$	1	1	1	1	±1	±1	±1	±1
$(0B0\pm)={}_0B_0^{\pm}$	1	1	-1	-1	±1	±1	∓1	∓1
$(0Em\pm)={}_0E_{m,-m}^{\pm}$	$2\cos 2rm\alpha$	$2\cos(2r+1)m\alpha$	0	0	$\pm2\cos 2rm\alpha$	$\pm2\cos(2r+1)m\alpha$	0	0
$(0Aq\pm)={}_0A_q^{\pm}$	1	-1	1	-1	±1	∓1	±1	∓1
$(0Bq\pm)={}_0B_q^{\pm}$	1	-1	-1	1	±1	∓1	∓1	±1
$(kEA0)={}_k^{-k}E_{A_0}$	$2\cos kt$	$2\cos k(\frac{1}{2}+t)$	$2\cos kt$	$2\cos k(\frac{1}{2}+t)$	0	0	0	0
$(kEB0)={}_k^{-k}E_{B_0}$	$2\cos kt$	$2\cos k(\frac{1}{2}+t)$	$-2\cos kt$	$-2\cos k(\frac{1}{2}+t)$	0	0	0	0
$(kGm)={}_k^{-k}G_{m,-m}$	$4\cos kt\cos 2mr\alpha$	$4\cos k(\frac{1}{2}+t)$ $\times\cos(2r+1)m\alpha$	0	0	0	0	0	0
$(kEAq)={}_k^{-k}E_{A_q}$	$2\cos kt$	$-2\cos k(\frac{1}{2}+t)$	$2\cos kt$	$-2\cos k(\frac{1}{2}+t)$	0	0	0	0
$(kEBq)={}_k^{-k}E_{B_q}$	$2\cos kt$	$-2\cos k(\frac{1}{2}+t)$	$-2\cos kt$	$2\cos k(\frac{1}{2}+t)$	0	0	0	0
$(\pi EA)={}_\pi E_{A_0}^{A_2}$	$2(-1)^t$	0	$2(-1)^t$	0	0	0	0	0
$(\pi EB)={}_\pi E_{B_0}^{B_2}$	$2(-1)^t$	0	$-2(-1)^t$	0	0	0	0	0
$(\pi Gj)={}_\pi G_{j,-j}^{\bar{j},-\bar{j}}$	$(-1)^t 4\cos 2jr\alpha$	0	0	0	0	0	0	0
amd only for $q=2h=2,4,6\ldots$								
$(\pi Eh\pm)={}_\pi E_{h,-h}^{\pm}$	$2(-1)^{r+t}$	0	0	0	0	0	$\pm2(-1)^{t+r}$	0

Table 6. Decomposition of the Kronecker products of reps of $L(2q)_q/mcm$ $(n=2q=2,4\ldots)$. The rep symbols are defined in table 5.

$\mathbf{(0A0+)}\times(0A0+)=(0A0+)$
$\mathbf{(0A0-)}\times(0A0\pm)=(0A0\mp)$
$\mathbf{(0B0+)}\times(0A0\pm)=(0B0\pm)$ $\quad (0B0+)\times(0B0+-=(0A0+)$

$\mathbf{(0B0-)}\times(0A0\pm)=(0B0\mp)$ $\quad (0B0-)\times(0B0\pm)=(0A0\mp)$
$\mathbf{(0Em+)}\times(0A0\pm)=(0Em+)\times(0B0\pm)=(0Em\pm)$
$(0Em+)\times(0Em'+)=$ (1) $(0Ev+)+(0Ew+)$ $\quad$ (2) $(0A0+)+(0B0+)+(0Ew+)$
(3) $(0Ev+)+(0Aq+)+(0Bq+)$ $\quad$ (4) $(0A0+)+(0B0+)+(0Aq+)+(0Bq+)$

$\mathbf{(0Em-)}\times(0A0\pm)=(0Em-)\times(0B0\pm)=(0Em\mp)$
$(0Em-)\times(0Em'\pm)=$ (1) $(0Ev\mp)+(0Ew\mp)$ $\quad$ (2) $(0A0\mp)+(0B0\mp)+(0Ew\mp)$
(3) $(0Ev\mp)+(0Aq\mp)+(0Bq\mp)$ $\quad$ (4) $(0A0\mp)+(0B0\mp)+(0Aq\mp)+(0Bq\mp)$

$\mathbf{(0Aq+)}\times(0A0\pm)=(0Aq\pm)$ $\quad (0Aq+)\times(0B0\pm)=(0Bq\pm)$ $\quad (0Aq+)\times(0Em\pm)=(0Eu\pm)$
$(0Aq+)\times(0Aq+)=(0A0+)$

$\mathbf{(0Aq-)}\times(0A0\pm)=(0Aq\mp)$ $\quad (0Aq-)\times(0B0\pm)=(0Bq\mp)$ $\quad (0Aq-)\times(0Em\pm)=(0Eu\mp)$
$(0Aq-)\times(0Aq\pm)=(0A0\mp)$

$\mathbf{(0Bq+)}\times(0A0\pm)=(0Bq\pm)$ $\quad (0Bq+)\times(0B0\pm)=(0Aq\pm)$ $\quad (0Bq+)\times(0Em\pm)=(0Eu\pm)$
$(0Bq+)\times(0Aq\pm)=(0B0\pm)$ $\quad (0Bq+)\times(0Bq+)=(0A0+)$

$\mathbf{(0Bq-)}\times(0A0\pm)=(0Bq\mp)$ $\quad (0Bq-)\times(0B0\pm)=(0Aq\mp)$ $\quad (0Bq-)\times(0Em\pm)=(0Eu\mp)$
$(0Bq-)\times(0Aq\pm)=(0B0\mp)$ $\quad (0Bq-)\times(0Bq\pm)=(0A0\mp)$

$\mathbf{(kEA0)}\times(0A0\pm)=(kEA0)$ $\quad (kEA0)\times(0B0\pm)=(kEB0)$ $\quad (kEA0)\times(0Em\pm)=(kGm)$
$(kEA0)\times(0Aq\pm)=(kEAq)$ $\quad (kEA0)\times(0Bq\pm)=(kEBq)$
$(kEA0)\times(k'EA0)=$ (e) $(xEA0)+(yEA0)$ $\quad$ (f) $(xEA0)+(yEAq)$
(g) $(0A0+)+(0A0-)+(yEA0)$ $\quad$ (h) $(0A0+)+(0A0-)+(yEAq)$
(c) $(xEA0)+(\pi EA)$ $\quad$ (d) $(0A0+)+(0A0-)+(\pi EA)$

$\mathbf{(kEB0)}\times(0A0\pm)=(kEB0)$ $\quad (kEB0)\times(0B0\pm)=(kEA0)$ $\quad (kEB0)\times(0Em\pm)=(kGm)$
$(kEB0)=(0Aq\pm)=(kEBq)$ $\quad (kEB0)\times(0Bq\pm)=(kEAq)$
$(kEB0)\times(k'EA0)=$ (e) $(xEB0)+(yEB0)$ $\quad$ (f) $(xEB0)+(yEBq)$
(g) $(0B0+)+(0B0-)+(yEB0)$ $\quad$ (h) $(0B0+)+(0B0-)+(yEBq)$
(c) $(xEB0)+(\pi EB)$ $\quad$ (d) $(0B0+)+(0B0-)+(\pi EB)$
$(kEB0)\times(k'EB0)=$ (e) $(xEA0)+(yEA0)$ $\quad$ (f) $(xEA0)+(yEAq)$
(g) $(0A0+)+(0A0-)+(yEA0)$ $\quad$ (h) $(0A0+)+(0A0-)+(yEAq)$
(c) $(xEA0)+(\pi EA)$ $\quad$ (d) $(0A0+)+(0A0-)+(\pi EA)$

$\mathbf{(kGm)}\times(0A0\pm)=(kGm)\times(0B0\pm)=(kGm)$
$(kGm)=(0Em'\pm)=$ (1) $(kGv)+(kGw)$ $\quad$ (2) $(kEA0)+(kEB0)+(kGw)$
(3) $(kGv)+(kEAq)+(kEBq)$ $\quad$ (4) $(kEA0)+(kEB0)+(kEAq)+(kEBq)$
$(kGm)\times(0Aq\pm)=(kGm)+(0Bq\pm)=(kGu)$
$(kGm)\times(k'EA0)=(kGm)\times(k'EB0)=$
(e) $(xGm)+(yGm)$ $\quad$ (f) $(xGm)+(yGu)$
(g) $(0Em+)+(0Em-)+(yGm)$ $\quad$ (h) $(0Em+)+(0Em-)+(yGu)$
(c) $(xGm)+(\pi Gp)$ for $m\neq h$ and $(xGm)+(\pi Eh+)+(\pi Eh-)$ for $m=h$
(d) $(0Em+)+(0Em-)+(\pi Gp)$ for $m\neq h$ and $(0Em+)+(0Em-)+(\pi Eh+)+(\pi Eh-)$ for $m=h$
$(kGm)\times(k'Gm')=$
(e1) $(xGv)+(xGw)+(yGv)+yGw)$
(e2) $(xEA0)+(xEB0)+(xGw)+(yEA0)+(yEB0)+(yGw)$
(e3) $(xGv)+(xEAq)+(xEBq)+(yGv)+(yEAq)+(yEBq)$
(e4) $(xEA0)+(xEB0)+(xEAq)+(xEBq)+(yEA0)+(yEB0)+(yEAq)+(yEBq)$
(f1) $(xGv)+(xGw)+(yGq-v)+(yGq-w)$
(f2) $(xEA0)+(xEB0)+(xGw)+(yEAq)+(yEBq)+(yGq-w)$
(f3) $(xGv)+(xEAq)+(xEBq)+(yGq-v)+(yEA0)+(yEB0)$
(f4) $(xEA0)+(xEB0)+(xEAq)+(xEBq)+(yEA0)+(yEB0)+(yEAq)+(yEBq)$
(g1) $(0Ev+)+(0Ev-)+(0Ew+)+(0Ew-)+(yGv)+(yGw)$
(g2) $(0A0)+(0A0-)+(0B0+)+(0B0-)+(0Ew+)+(0Ew-)+(yEA0)+(yEB0)+(yGw)$

Table 6. (continued)

(g3) $(0Ev+)+(0Ev-)+(xEAq)+(xEBq)+(yGv)+(yEAq)+(yEBq)$
(g4) $(0A0+)+(0A0-)+(0B0+)+(0B0-)+(0Aq+)+(0Aq-)+(0Bq+)+(0Bq-)+(yEA0)+(yEB0)$ $+(yEAq)+(yEBq)$
(h1) $(0Ev+)+(0Ev-)+(0Ew+)+(0Ew-)+(yG\,q-w)+(yG\,q-v)$
(h2) $(0A0+)+(0A0-)+(0B0+)+(0B0-)+(0Ew+)+(0Ew-)+(yEAq)+(yEBq)+(yG\,q-w)$
(h3) $(0Ev+)+(0Ev-)+(xEAq)+(xEBq)+(yG\,q-v)+(yEA0)+(yEB0)$
(h4) $(0A0+)+(0A0-)+(0B0+)+(0B0-)+(0Aq+)+(0Aq-)+(0Bq+)+(0Bq-)+(yEAq)+(yEBq)$ $+(yEA0)+(yEB0)$
(c1) $(xGv)+(xGw)+(\pi Gp_1)+(\pi Gp_2)$ for $m+m'\neq h, 3h$ and $m-m'\neq \pm h$
$(xGv)+(xGw)+(\pi Eh+)+(\pi Eh-)+(\pi Gp_2)$ for $m+m'=h, 3h$ and $m-m'\neq \pm h$
$(xGv)+(xGw)+(\pi Gp_1)+(\pi Eh+)+(\pi Eh-)$ for $m+m'\neq h, 3h$ and $m-m'=\pm h$
(c2) $(xEA0)+(xEB0)+(xGw)+(\pi EAq)+(\pi EBq)+(\pi Gp_1)$ for $m+m'\neq h, 3h$
$(xEA0)+(xEB0)+(xGw)+(\pi EAq)+(\pi EBq)+(\pi Eh+)+(\pi Eh-)$ for $m+m'=h, 3h$
(c3) $(xGv)+(xEAq)+(xEBq)+(\pi Gp_1)+(\pi EA)+(\pi EB)$ for $m-m'\neq \pm h$
$(xGv)+(xEAq)+(xEBq)+(\pi Eh+)+(\pi Eh-)+(\pi EA)+(\pi EB)$ for $m-m'=\pm h$
(c4) $(xEA0)+(xEB0)+(xEAq)+(xEBq)+2(\pi EA)+2(\pi EB)$
(d1) $(0Ev+)+(0Ev-)+(0Ew+)+(0Ew-)+(\pi Gp_1)+(\pi Gp_2)$ for $m+m'\neq h, 3h$ and $m-m'\neq \pm h$
$(0Ev+)+(0Ev-)+(0Ew+)+(0Ew-)+(\pi Eh+)+(\pi Eh-)+(\pi Gp_2)$ for $m+m'=h, 3h$ and $m-m'\neq \pm h$
$(0Ev+)+(0Ev-)+(0Ew+)+(0Ew-)+(\pi Gp_1)+(\pi Eh+)+(\pi Eh-)$ for $m+m'\neq h, 3h$ and $m-m'=\pm h$
(d2) $(0A0+)+(0A0-)+(0B0+)+(0B0-)+(0Ew+)+(0Ew-)+(\pi EAq)+(\pi EBq)+(\pi Gp_1)$ for $m+m'\neq h, 3h$
$(0A0+)+(0A0-)+(0B0+)+(0B0-)+(0Ew+)+(0Ew-)+(\pi Eq1)+(\pi EBq)+(\pi Eh+)+(\pi Eh-)$ for $m+m'=h, 3h$
(d3) $(0Ev+)+(0Ev-)+(0Aq+)+(0Aq-)+(0Bq+)+(0Bq-)+(\pi Gp_2)+(\pi EAq)+(\pi EBq)$ for $m-m'\neq \pm h$
$(0Ev+)+(0Ev-)+(0Aq+)+(0Aq-)+(0Bq+)+(0Bq-)+(\pi Eh+)+(\pi Eh-)+(\pi EAq)+(\pi EBq)$ for $m-m'=\pm h$
(d4) $(0A0+)+(0A0-)+(0B0+)+(0B0-)+(0Aq+)+(0Aq-)+(0Bq+)+(0Bq-)+2(\pi EAq)+2(\pi EBq)$

(kEA) $\times(0A0\pm)=(kEAq)$ $(kEAq)+(0B0\pm)=(kEBq)$ $(kEAq)\times(0Em\pm)=(kGu)$
$(kEA)\times(0Aq\pm)=(kEA0)$ $(kEAq)\times(0Bq\pm)=(kEB0)$

$(kEAq)\times(k'EA0)=$	(e) $(xEAq)+(yEAq)$	(f) $(xEAq)+(yEA0)$
	(g) $(0Aq+)+(0Aq-)+(yEAq)$	(h) $(0Aq+)+(0Aq-)+(yEA0)$
	(c) $(xEAq)+(\pi EA)$	(d) $(0Aq+)+(0Aq-)+(\pi EA)$
$(kEAq)\times(k'EB0)=$	(e) $(xEBq)+(yEBq)$	(f) $(xEBq)+(yEA0)$
	(g) $(0Bq+)+(0Bq-)+(yEBq)$	(h) $(0Bq+)+(0Bq-)+(yEB0)$
	(c) $(xEBq)+(\pi EB)$	(d) $(0Bq+)+(0Bq-)+(\pi EB)$
$(kEAq)\times(k'Gm)=$	(e) $(xGu)+(yGu)$	(f) $(xGu)+(yGm)$
	(g) $(0Eu+)+(0Eu-)+(yGu)$	(h) $(0Eu+)+(0Eu-)+(yGm)$
	(c) $(xGu)+(\pi Gp)$ for $m\neq h$ and $(xGu)+(\pi Eh+)+(\pi Eh-)$ for $m=h$	
	(d) $(0Eu+)+(0Eu-)+(\pi Gp)$ for $m\neq h$ and $(0Eu+)+(0Eu-)+(\pi Eh+)+(\pi Eh-)$ for $m=h$	
$(kEAq)\times(k'EAq)=$	(e) $(xEA0)+(yEA0)$	(f) $(xEA0)+(yEAq)$
	(g) $(0A0+)+(0A0-)+(yEA0)$	(h) $(0A0+)+(0A0-)+(yEAq)$
	(c) $(xEA0)+(\pi EA)$	(d) $(0A0+)+(0A0-)+(\pi EA)$

$(kEBq)$ $\times(0A0\pm)=(kEBq)$ $(kEBq)+(0B0\pm)=(kEAq)$ $(kEBq)+(0Em\pm)=(kGu)$
$(kEBq)\times(0Aq\pm)=(kEB0)$ $(kEBq)\times(0Bq\pm)=(kEA0)$

$(kEBq)\times(k'EA0)=$	(e) $(xEBq)+(yEBq)$	(f) $(xEBq)+(yEB0)$
	(g) $(0B+)+(0Bq-)+(yEBq)$	(h) $(0Bq+)+(0Bq-)+(yEB0)$
	(c) $(xEBq)+(\pi EB)$	(d) $(0Bq+)+(0Bq-)+(\pi EB)$
$(kEBq)\times(k'EB0)=$	(e) $(xEAq)+(yEAq)$	(f) $(xEAq)+(yEA0)$
	(g) $(0Aq+)+(0Aq-)+(yEAq)$	(h) $(0Aq+)+(0Aq-)+(yEA0)$
	(c) $(xEAq)+(\pi EA)$	(d) $(0Aq+)+(0Aq-)+(\pi EA)$

Table 6. (continued)

$(kEBq)\times(k'Gm)=$	(e) $(xGu)+(yGu)$	(f) $(xGu)+(yGm)$
	(g) $(0Eu+)+(0Eu-)+(yGu)$	(h) $(0Eu+)+(0Eu-)+(yGm)$
	(c) $(xGu)+(\pi Gp)$ for $m\neq h$ and $(xGu)+(\pi Eh+)+(\pi Eh-)$ for $m=h$	
	(d) $(0Eu+)+(0Eu-)+(\pi Gp)$ for $m\neq h$ and $(0Eu+)+(0Eu-)+(\pi Eh+)$ $+(\pi Eh-)$ for $m=h$	
$(kEBq)\times(k'EAq)=$	(e) $(xEB0)+(yEB0)$	(f) $(xEB0)+(yEBq)$
	(g) $(0B0+)+(0B0-)+(yEB0)$	(h) $(0B0+)+(0B0-)+(yEBq)$
	(c) $(xEB0)+(\pi EB)$	(d) $(0B0+)+(0B0-)+(\pi EB)$
$(kEBq)\times(k'EBq)=$	(e) $(xEA0)+(yEA0)$	(f) $(xEA0)+(yEAq)$
	(g) $(0A0+)+(0A0-)+(yEA0)$	(h) $(0A0+)+(0A0-)+(yEAq)$
	(c) $(xEA0)+(\pi EA)$	(d) $(0A0+)+(0A0-)+(\pi EA)$

$\boldsymbol{(\pi EA)}\times(\pi EA)$ $\quad(\pi EA)\times(0B0\pm)=(\pi EB)$
$(\pi EA)\times(0Em\pm)=(\pi Gp)$ for $m\neq h$ and $(\pi Eh+)+(\pi Eh-)$ for $m=h$
$(\pi EA)\times(0Aq\pm)=(\pi EA)$ $\quad(\pi EA)\times(0Bq\pm)=(\pi EB)$ $\quad(\pi EA)\times(kEA0)=(zEA0)+(zEAq)$
$(\pi EA)\times(kEB0)=(zEB0)+(zEBq)$ $\quad(\pi EA)\times(kGm)=2(zGm)$
$(\pi EA)\times(kEAq)=(zEA0)+(zEAq)$ $\quad(\pi EA)\times(kEBq)=(zEB0)+(zEBq)$
$(\pi EA)\times(\pi EA)=(0A0+)+(0A0-)+(0Aq+)+(0Aq-)$

$\boldsymbol{(\pi EB)}\times(0A0\pm)=(\pi EB)$ $\quad(\pi EB)\times(0B0\pm)=(\pi EA)$
$(\pi EB)\times(0Em\pm)=(\pi Gp)$ for $m\neq h$ and $(\pi Eh+)+(\pi Eh-)$ for $m=h$
$(\pi EB)\times(0Aq\pm)=(\pi EB)$ $\quad(\pi EB)\times(0Bq\pm)=(\pi EA)$ $\quad(\pi EB)\times(kEA0)=(zEB0)+(zEBq)$
$(\pi EB)\times(kEB0)=(zEA0)+(zEAq)$ $\quad(\pi EB)\times(kGm)=2(zGm)$
$(\pi EB)\times(kEAq)=(zEB0)+(zEBq)$ $\quad(\pi EB)\times(kEBq)=(zEA0)+(zEAq)$
$(\pi EB)\times(\pi EA)=(0B0+)+(0B0-)+(0Bq+)+(0Bq-)$
$(\pi EB)\times(\pi EB)=(0A0+)+(0A0-)+(0Aq+)+(0Aq-)$

$\boldsymbol{(\pi Gj)}\times(0A0\pm)=(\pi Gj)\times(0B0\pm)=(\pi Gj)$
$(\pi Gj)\times(0Em\pm)=$
$(\pi G\beta)+(\pi G\gamma)$ for $j-m\neq -h, 0$ and $j+m\neq h, q$
$(\pi Eh+)+(\pi Eh-)+(\pi G\gamma)$ for $j-m=-h$ and $j+m\pm h, q$
$(\pi EA)+(\pi EB)+(\pi G\gamma)$ for $j-m=0$ and $j+m\neq h, q$
$(\pi G\beta)+(\pi Eh+)+(\pi Eh-)$ for $j-m\neq -h, 0$ and $j+m=h$
$(\pi G\beta)+(\pi EA)+(\pi EB)$ for $j-m\neq -h, 0$ and $j+m=q$
$(\pi Eh+)+(\pi Eh-)+(\pi EA)+(\pi EB)$ for $j-m=-h$ and $j+m=q$, or $j-m=0$ and $j+m=h$
$(\pi Gj)\times(0Aq\pm)=(\pi Gj)\times(0Bq\pm)=(\pi Gj)$
$(\pi Gj)\times(kEA0)=(\pi Gj)\times(kEB0)=(zGj)+(zG\kappa)$
$(\pi Gj)\times(kGm)=$
$(zG\delta)+(zG\varepsilon)+(zG\eta)+(zG\zeta)$ for $j\neq m, q-m$
$(zEA0)+(zEB0)+(zG\varepsilon)+(zEAq)+(zEBq)+(zG\zeta)$ for $j=m$ and $j+m\neq q$
$(zG\delta)+(zEAq)+(zEBq)+(zG\eta)+(zEA0)+(zEB0)$ for $j\neq m$ and $j+m=q$
$(\pi Gj)\times(kEAq)=(\pi Gj)\times(kEBq)=(zGj)+(zG\kappa)$
$(\pi Gj)\times(\pi EA)=(\pi Gj)\times(\pi EB)=(0Ej+)+(0Ej-)+(0E\kappa+)+(0E\kappa-)$
$(\pi Gj)\times(\pi Gj')=$
$(0E\lambda+)+(0E\lambda-)+(0E\mu+)+(0E\mu-)+(0E\nu+)+(0E\nu-)+(0E\varphi+)+(0E\varphi-)$ for $j\neq j'$
$(0A0+)+(0A0-)+(0B0+)+(0B0-)+(0E\mu+)+(0E\mu-)+(0Aq+)+(0Aq-)+(0Bq+)$
$+(0Bq-)+(0E\varphi+)+(0E\varphi-)$ for $j=j'$

$\boldsymbol{(\pi Eh+)}\times(0A0\pm)=(\pi Eh+)\times(0B0\mp)=(\pi Eh\pm)$
$(\pi Eh+)\times(0Em\pm)=(\pi Gp)$ for $m\neq h$ and $(\pi EA)+(\pi EB)$ for $m=h$
$(\pi Eh+)\times(0Aq\pm)=(\pi Eh+)\times(0Bq\mp)=(\pi Eh\pm)$
$(\pi Eh+)\times(kEA0)=(\pi Eh+)\times(kEB0)=(zGh)$
$(\pi Eh+)\times(kGm)=(zG\rho)+(zG\sigma)$ for $m\neq h$ and $(zEA0)+(zEB0)+(zEAq)+(zEBq)$ for $m=h$
$(\pi Eh+)\times(kEAq)+(\pi Eh+)\times(kEBq)=(zGh)$
$(\pi Eh+)\times(\pi EA)=(\pi Eh+)\times(\pi EB)=(0Eh+)+(0Eh-)$
$(\pi Eh+)\times(\pi Gj)=(0E\tau+)+(0E\tau-)+(0E\psi+)+(0E\psi-)$
$(\pi Eh+)\times(\pi Eh+)=(0A0+)+(0B0-)+(0Aq+)+(0Bq-)$

Table 6. (continued)

$(\pi Eh-)\times(0A0\pm)=(\pi Eh-)\times(0B0\mp)=(\pi Eh\mp)$
$(\pi Eh-)\times(0Em\pm)=(\pi G\rho)$ for $m\neq h$ and $(\pi EA)+(\pi EB)$ for $m=h$
$(\pi Eh-)\times(0Aq\pm)=(\pi Eh-)\times(0Bq\mp)=(\pi Eh\mp)$
$(\pi Eh-)\times(kEA0)=(\pi Eh-)\times(kEB0)=(zGh)$
$(\pi Eh-)\times(kGm)=(zG\rho)+(zG\sigma)$ for $m\neq h$ and $(zEA0)+(zEB0)+(zEAq)+(zEBq)$ for $m=h$
$(\pi Eh-)\times(kEAq)=(\pi Eh-)\times(kEBq)=(zGh)$
$(\pi Eh-)\times(\pi EA)=(\pi Eh-)\times(\pi EB)=(0Eh+)+(0Eh-)$
$(\pi Eh-)\times(\pi Gj)=(0E\tau+)+(0E\tau-)+(0E\psi+)+(0E\psi-)$
$(\pi Eh-)\times(\pi Eh\pm)=(0A0\mp)+(0B0\pm)+(0Aq\mp)+(0Bq\pm)$

4. Conclusions and discussions

The selection rules given in tables 2, 4 and 6 imply the following conservation laws.

The quasi-momentum is conserved for all the line groups considered here, in the following sense:

$$k_f \doteq \pm(k_i \pm k_v),$$

where i, f and v label the initial state, the final state and the perturbation, respectively, and where '$\doteq$' means 'equal mod 2π.' Namely, each of the line groups under study contains $(\sigma_h|0)$ which reverses the quasi-momentum, and hence the energy eigenvalues are labelled by pairs $\{k, -k\}$ here, in general.

The quasi-angular momentum is strictly conserved in the case of line groups $L(\overline{2n})2m$ and $L(\overline{2n})2c$ (n odd), Ln/mmm and Ln/mcc (n even), in an analogous way:

$$m_f \doteq \pm(m_i \pm m_v),$$

where '$\doteq$' means 'equal mod n'. On the other hand, the line group $L(2q)_q/mcm$ contains $(C_{2q}|\frac{1}{2})$ rather than $(C_{2q}|0)$. Fortunately, this non-symmorphic line group can be embedded into a symmorphic line group containing $(C_{2q}|0)$ as an element, so that the quantum label m can be retained. However, the consequence is that m jumps by $\pm q$ in Umklapp processes.

The parity with respect to the horizontal mirror plane σ_h (denoted by + or − in the rep symbol) is strictly conserved in all the line groups isogonal to D_{nh}.

The parity with respect to the vertical mirror plane σ_v (distinguished by A and B for even and odd states, respectively) is always conserved in the $L(\overline{2n})2m$ (n odd) and Ln/mmm, $L(2q)_q/mcm$ (n even) groups. The line groups $L(\overline{2n})2c$ (n odd) and Ln/mcc (n even) do not contain the element $(\sigma_v|0)$ but $(\sigma_v|\frac{1}{2})$, and therefore in these cases an A state is mapped onto an A state in normal processes, but onto a B state in Umklapp processes (the perturbation is assumed to be of A type).

The reps of line groups isogonal to D_{nh} are all of the Frobenius-Schur type (a) (Božović and Božović 1981) and hence the addition of time reversal to these groups does not affect the selection rules given above.

A general remark concerning this whole series is that triple and higher Kronecker products can now be easily decomposed into irreducible constituents, since this product is distributive relative to the direct sum of reps. Also, some further selection rules can be obtained through the Wigner-Eckart theorem, when multidimensional reps are involved. In fact, the Clebsch-Gordan coefficients (Hamermesh 1962) can be uniquely determined for all the line groups except for the families $L\tilde{n}c$, $L(\overline{2n})2c$ (n odd) and

$L(\overline{2n})2m$, $L(\overline{2n})2c$, Ln/mcc, $L(2q)_q/mcm$ (n even), when in the Kronecker products of two four-dimensional reps some frequency coefficients are equal to 2. Further, the families $Ln2$, Ln_p2, $L\bar{n}m$, $L(\overline{2n})2m$ (n odd) and $Ln22$, Ln_p22, Ln/mmm (n even) consist of simply reducible groups. Finally, note that these tables include (for $k=0$) the selection rules for all the axial point groups (C_n, C_{nv}, C_{nh}, S_{2n}, D_n, D_{nd} and D_{nh}) given in physically meaningful and mnemonic language of quasi-angular momentum and parities.

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